The Brilliant Cut

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1. Introduction

The brilliant is the most common diamond cut. Its optical appearance depends on the parameters of its different parts. The price of a polished diamond mainly depends on the weight, the color, the clarity and the cut, commonly known as the 4 C's. The goal in diamond polishing today is to reach both a high optical performance and a high yield. This brings us to the diamond polisher who basically needs a work plan that tells him how to achieve this goal. It all ends up with the calculation of the necessary parameters of the brilliant cut.

The present paper is the first in a series on the technical aspects of diamond manufacturing.

1.1 Description of the brilliant cut

Part	Parameter	Symbol	Unit
	Diameter	$\varnothing$	mm
	Total Depth	H_t	%
Crown			
	Table diameter	$\varnothing_t$	%
	Height	H_c	%
	Bezel Angle	β_c	°
	Upper-half length	p	ratio
	Upper-half angle	β_h	°
	Star angle	β_s	°
	Star top angle	γ_s	°
Girdle			
	Height at bezels	H_g	%
	Height at half facets	H_{gmin}	%
Pavilion			
	Culet	$\varnothing_c$	%
	Depth	H_p	%
	Angle	α_p	°
	Lower-half length	q	ratio
	Lower-half angle	α_h	°

Table 1: Parameters of the Brilliant Cut

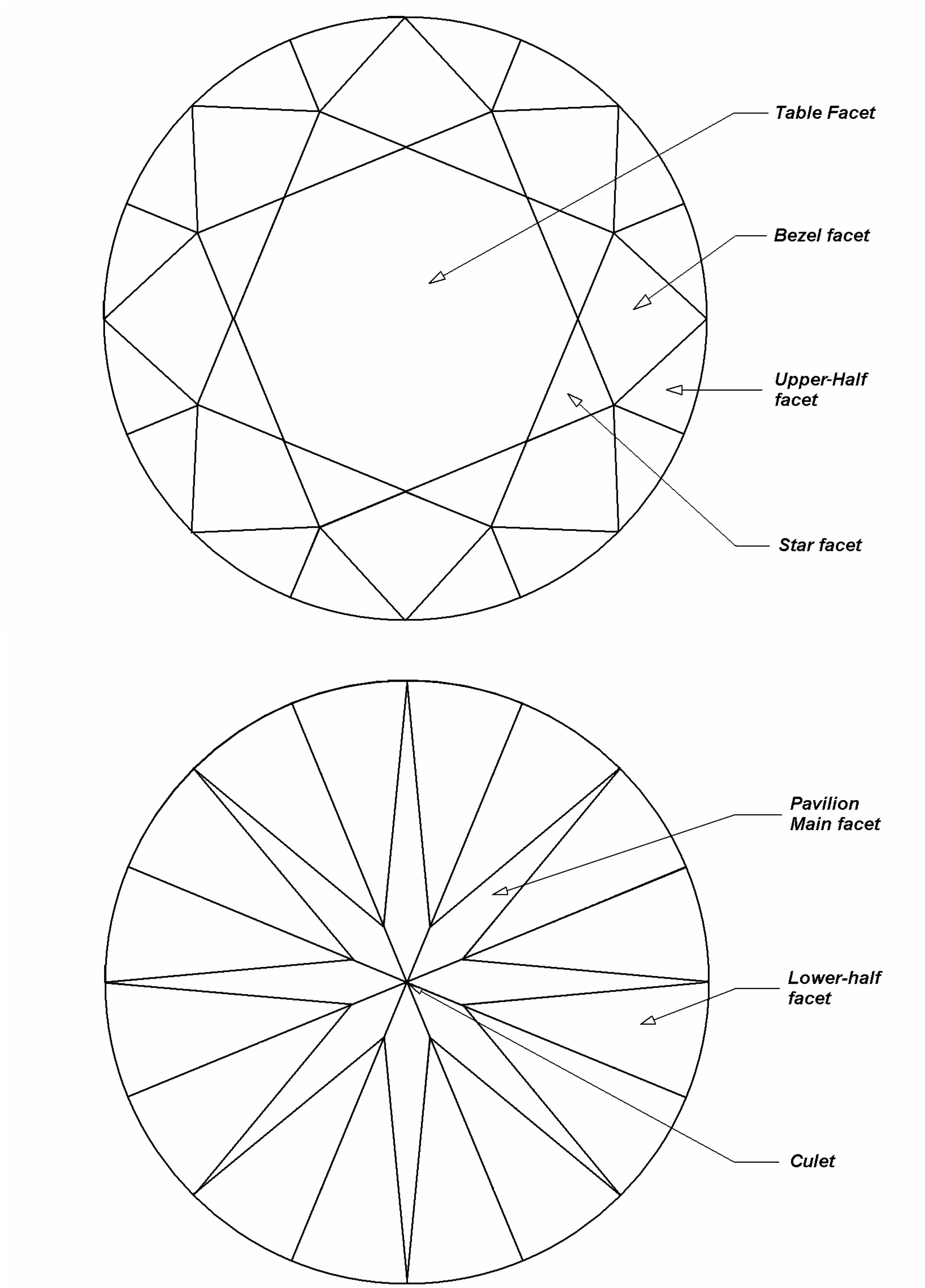


Figure 1: The facets of the brilliant cut

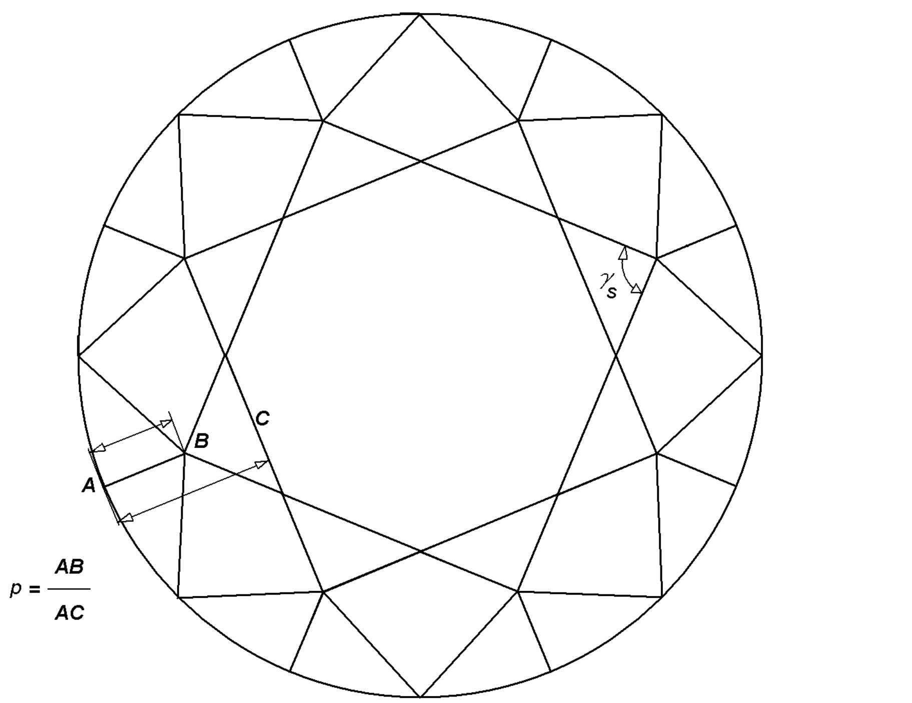


Figure 2: Upper-half length p and top star angle γ_s

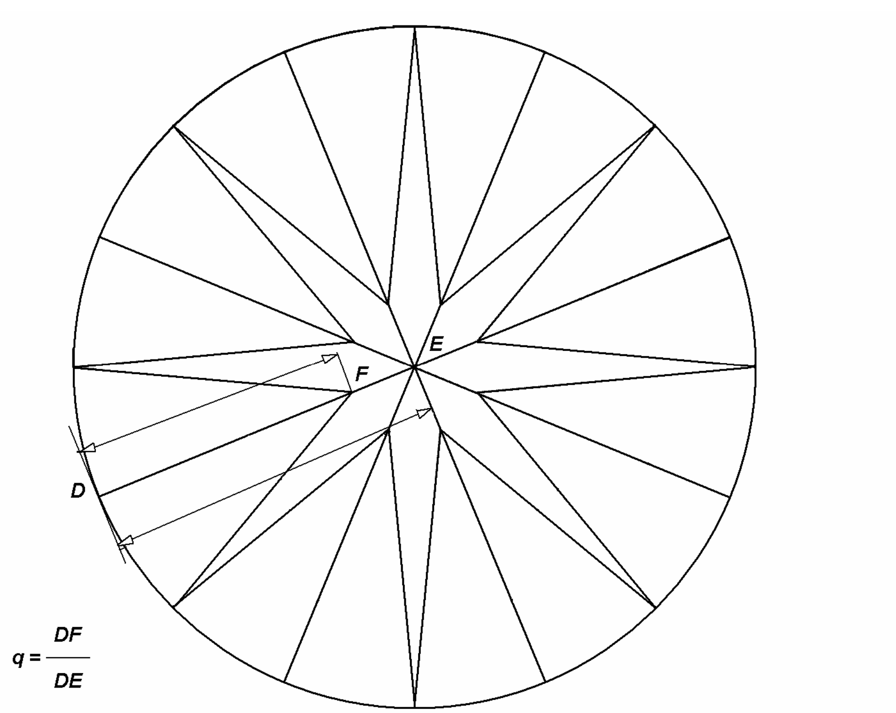


Figure 3: Lower-half length *q*

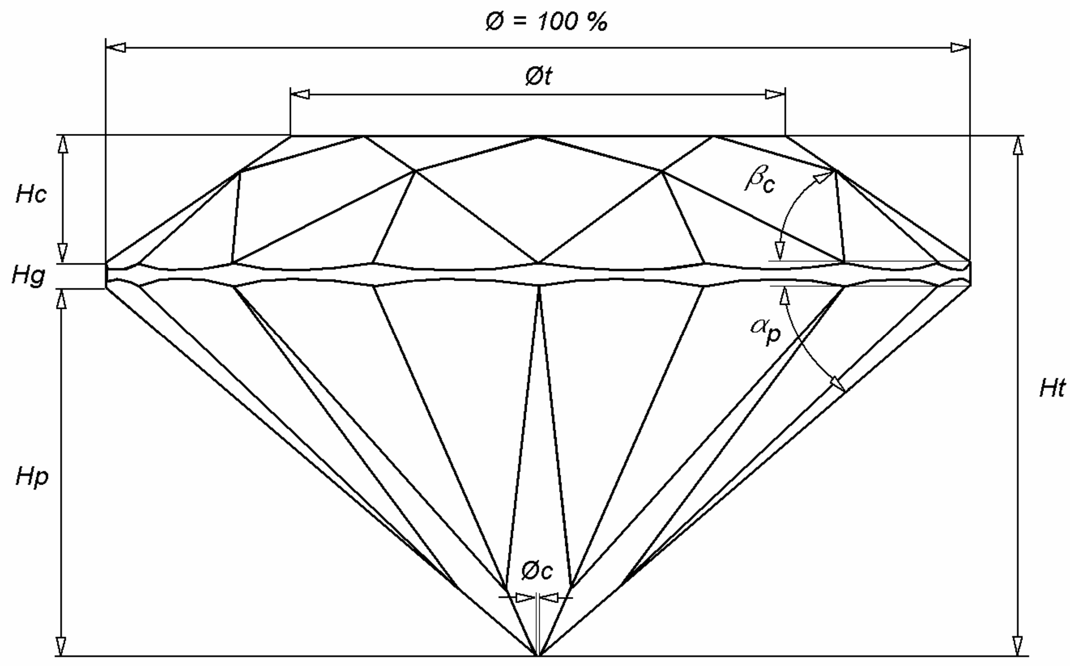
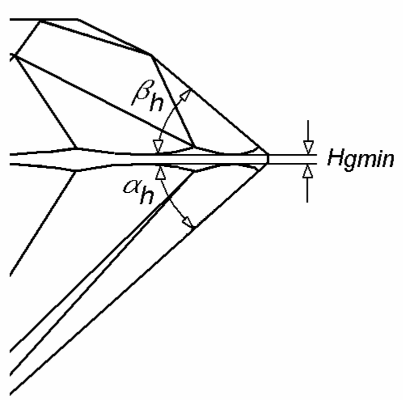


Figure 4: Front Side



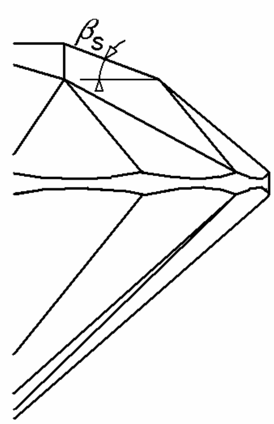


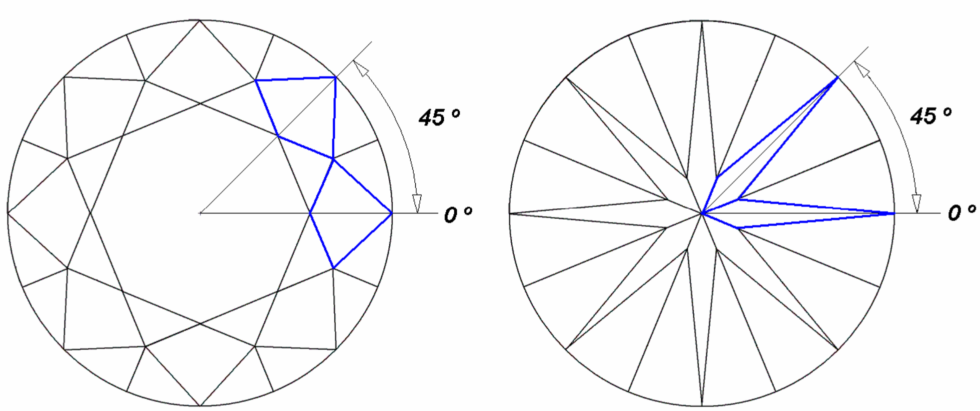
Figure 5: Half angle and minimum girdle

Figure 6: Star Angle

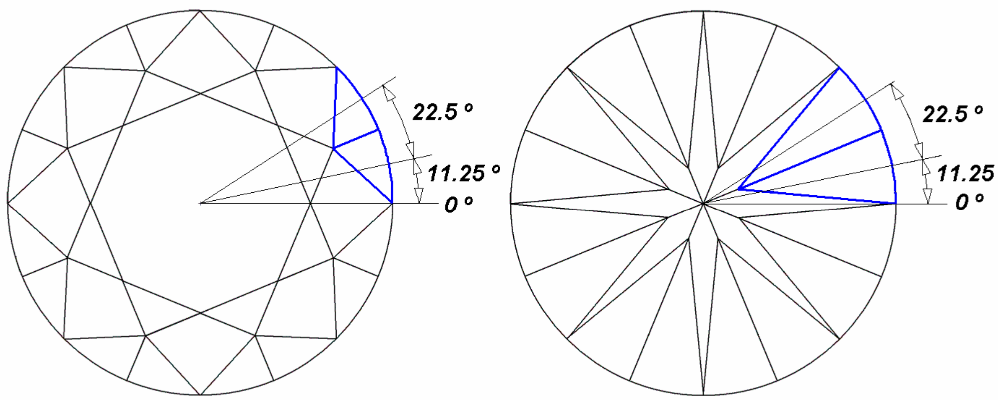
1.2 Assumptions

The formulas presented here are valid for a perfect brilliant. This means:

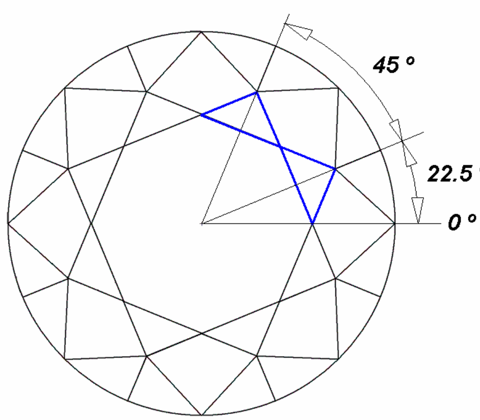
1. The reference for a brilliant is the diameter (Ø) of the girdle. All other length measurements are relative to this diameter. In all formulas the parameters relative to the diameter have a value between 0 and 100 divided by 100. For instance a table diameter of 58% is represented as 0.58. To calculate the absolute value of a relative parameter according to a specific diameter Ø, you multiply the relative value with Ø: For instance if the diameter is 10 mm then a table diameter of 58% becomes 10 x 0.58 = 5.8 mm.
2. All the angular parameters are expressed in radians between 0 and 2π.
3. The brilliant is perfectly symmetrical. This means that using any of the formulas on real live examples can give deviations because of slight variations from perfect symmetry.
4. The brilliant has no external characteristics, like small holes or unpolished surfaces of the original rough, left on the diamond for weight retention.
5. The girdle is circular and not facetted.
6. The crown bezels are aligned with the pavilion main facets.
7. The direction of the bezel and pavilion facets is a multiple of 45º (π/4 rad) starting at zero.



8. The direction of the upper- and lower-half facets is a multiple of 22.5º (π/8 rad) starting at 11.25º (π/16 rad).



9. The direction of the star facets is a multiple of 45º (π/4 rad) starting at 22.5º (π/8 rad)



2 The Crown Properties

The crown has four main parameters as shown in figures 2 and 4

- The Table width ($\varnothing_t$)
- The Crown Height (H_c)
- The Bezel Angle (β_c)
- The upper-half length (p)

The parameters $\varnothing_t$, H_c and β_c are related. Only two parameters can be freely defined, the third one can then be calculated. The relationship between the parameters is defined in formulas 1.1, 1.2 and 1.3.

$$H_c = \frac{(1.0 - \varnothing_t) tg\beta_c}{2} \tag{1.1}$$

$$tg\beta_c = \frac{2H_c}{1 - \varnothing_t} \tag{1.2}$$

$$\varnothing_t = 1 - \frac{2H_c}{tg\beta_c} \tag{1.3}$$

$$p = \frac{AB}{AC} \tag{1.4}$$

The parameters for the additional upper-half and star facets, are defined as:

- The upper-half angle (β_h)
- The star angle (β_s)
- The top star angle (γ_s)

The halve angle (β_h) is defined by:

$$tg\beta_h = tg\beta_c \frac{1-\left(1-p\left(1-\varnothing_t \cos\frac{\pi}{8}\right)\right)\cos\frac{\pi}{8}}{p\left(1-\varnothing_t \cos\frac{\pi}{8}\right)\cos\frac{\pi}{16}} \qquad (1.5)$$

The star angle (β_s) is defined by:

$$tg\beta_s = tg\beta_c \frac{\left(1-p\left(1-\varnothing_t \cos\frac{\pi}{8}\right)\right)\cos\frac{\pi}{8}-\varnothing_t}{(1-p)\left(1-\varnothing_t \cos\frac{\pi}{8}\right)} \qquad (1.6)$$

The top angle of the stars (γ_s) is defined by:

$$tg\left(\frac{\gamma_s}{2}\right) = \frac{\varnothing_t \sin\frac{\pi}{8}}{(1-p)\left(1-\varnothing_t \cos\frac{\pi}{8}\right)} \qquad (1.7)$$

3 The Girdle Properties

The girdle has two important parameters as shown in figures 4 and 5:

- The (maximum) girdle thickness (H_g)
- The minimum girdle thickness (H_{gmin})

The maximum thickness is reached at the edges of two upper- and lower-half facets and between a main crown facet and a pavilion facet. The minimum girdle is measured between the valleys of the upper- and lower-half facets.

The minimum girdle thickness can be calculated from the maximum girdle thickness (H_g) , the crown half angle (β_h) and the pavilion half angle (α_h).

$$H_{gmin} = H_g - \frac{\left(1-\cos\frac{\pi}{16}\right)\left(tg\beta_h + tg\alpha_h\right)}{2} \tag{1.8}$$

4 The Pavilion Properties

The pavilion has three main parameters as shown in figures 3, 4 and 5:

- The pavilion depth H_p
- The pavilion angle α_p
- The culet diameter $Ø_c$
- The lower-half length q

The lower-half facets are characterized by:

- The lower-half angle α_h

The main pavilion parameters are defined in equations 1.9, 1.10, 1.11 and 1.12. The culet diameter is typically very small. It is always the intention to keep the culet diameter as small as possible but, in certain cases, closing the culet would create an unacceptable weight loss. To be complete the culet diameter is incorporated into the equations. In most cases the simplified equations will be used for a culet diameter close to zero (pointed culet).

$$\varnothing_c = 1 - \frac{2H_p}{tg\alpha} \tag{1.9}$$

$$H_p = \frac{(1-\varnothing_c)\,tg\alpha}{2} \tag{1.10}$$

$$tg\alpha = \frac{2H_p}{1-\varnothing_c} \tag{1.11}$$

$$q = \frac{DF}{DE} \tag{1.12}$$

For $Ø_c = 0$ the equations 1.10 and 1.11 become:

$$H_p = \frac{tg\alpha}{2} \tag{1.13}$$

$$tg\alpha = 2H_p \tag{1.14}$$

The angle of the lower-half facets (α_h) can be calculated from the lower-half length q and the culet diameter ($\varnothing_c$).

$$tg\alpha_h = tg\alpha \frac{1-\left(1-q\left(1-\varnothing_c \cos\frac{\pi}{8}\right)\right)\cos\frac{\pi}{8}}{q\left(1-\varnothing_c \cos\frac{\pi}{8}\right)\cos\frac{\pi}{16}} \tag{1.15}$$

For $\varnothing_c=0$ this reduces to:

$$tg\alpha_h = tg\alpha \frac{1-(1-q)\cos\frac{\pi}{8}}{q\cos\frac{\pi}{16}} \tag{1.16}$$

5 The weight calculation of a brilliant cut

To calculate the weight of a brilliant, the volume is multiplied by the specific weight. For the calculation of the volume, the brilliant is split in three parts:

- The volume of the crown
- The volume of the girdle
- The volume of the pavilion

The volume of the crown (V_c) can be calculated from the formula (1.17):

$$V_c = \varnothing^3 \frac{2H_c}{3(1-\varnothing_t)} \sin\frac{\pi}{8}\left[1+\left(1-p\left(1-\varnothing_t \cos\frac{\pi}{8}\right)\right)\left(1-\cos\frac{\pi}{8}\right)-\varnothing_t^3 \cos\frac{\pi}{8}-\varnothing_t^2\left(\sin\frac{\pi}{8}\right)^2\left(1-p\left(1-\varnothing_t \cos\frac{\pi}{8}\right)\right)\right] \tag{1.17}$$

The volume of the girdle (V_g) can be calculated from the formula (1.18):

$$V_g = \varnothing^3\left[\frac{\pi H_g}{4}-\left(tg\beta_h + tg\alpha_h\right)\cos\frac{\pi}{16}\left(\frac{2}{3}tg\frac{\pi}{16}\left(3-\left(\sin\frac{\pi}{16}\right)^2\right)-\frac{\pi}{8}\right)\right] \tag{1.18}$$

The volume of the pavilion (V_p) can be calculated from the formula (1.19):

Considering that the culet is not pointed:

$$V_p = \varnothing^3 \frac{2}{3}\left[\frac{tg\alpha_p}{2}\sin\frac{\pi}{8}\left(1+\left(1-q\left(1-\cos\frac{\pi}{8}\right)\right)\right)-\left(\varnothing_c^3\left(\cos\frac{\pi}{8}\right)^2 \sin\frac{\pi}{8} tg\alpha_p\right)\right] \tag{1.19}$$

with a pointed culet ($Ø_c$=0) formula 1.19 transforms to:

$$V_p = \varnothing^3 \frac{2H_p}{3}\sin\frac{\pi}{8}\left[1+(1-q)\left(1-\cos\frac{\pi}{8}\right)\right] \tag{1.20}$$

The total volume V_{tot} (mm^3) is obtained from:

$$V_{tot} = V_c + V_g + V_p \qquad (1.21)$$

The weight of the diamond in carat (1 ct is 0.2 g) is calculated by multiplying the volume with the density (ρ) of the diamond (3.515 g/cm^3):

$$W = \rho . V_{tot} \qquad (1.22)$$

If the diameter *(Ø)* is expressed in mm, this leads to the weight in carats (0.2 g) :

$$W = 0.017575\ V_{tot} \qquad (1.23)$$